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PSEUDO-FOLLOWER REACTIVE FORCE AS A MEANS OF IMPROVING THE LOAD CARRYING CAPACITY OF STRUCTURAL SYSTEMS

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Abstract—This paper presents a method for improving the carrying capacity of structures due to the loss of overall structural stability. The proposed method is based on adding a displacement dependent force to a support according to a prescribed rule. Simple numerical examples are presented. These examples are confined to the application of the pseudo-follower reactive force and for in-plane framed structures. The results are obtained with the assumption that the change of the geometrical configuration due to loading is negligibly small before the instability occurs. The numerical results show that a significant improvement in the stability can be expected by the application of the proposed reaction system. In this paper only systems with non-conservative follower forces not explicitly dependent on time are considered.

NOTATION

E_{ijkl}	tensor of elastic moduli
e_{ij}	strain tensor
F_{cr}	carrying capacity
F_i	surface forces
F^0	nodal forces
$\mathbf{K}_{CF}$	corrective matrix for nodal follower loads
$\mathbf{K}_{CP}$	corrective matrix for distributed follower loads
$\mathbf{K}_{CR}$	corrective matrix for nodal reactive forces
$\mathbf{K}_E$	elastic stiffness matrix
$\mathbf{K}_G$	geometric stiffness matrix
L	length
$\mathbf{M}$	mass matrix
P_i	body forces
P^0	displacement-dependent distributed loads
Q^0	stress resultant
R_i	reactive forces
R^{pse}	pseudo-follower reactive forces
R^0	exceptionally conservative non-working reactive forces
t	time
u	displacement
V	volume
γ	multiplication factor of corrective force
δ_{ij}	Kronecker delta
λ	load multiplier
σ_{ij}	stress tensor
ϕ	rotation about axis
ϕ_{zi}	rotation about z axis at node i
ω	natural frequency
$(\ )_{,i}$	differentiation with respect to x_i, i.e. Lagrangian Cartesian coordinates $\partial(\ )/\partial x_i$
$(\ )^{non}$	parts and components concerned with displacement-dependence
$(\ )_{xi}, (\ )_{yi}$	x and y components of force applied at node i
$(\ )^0$	quantities under loading state.

1. INTRODUCTION

The purpose of this article is to propose a method which can improve the carrying capacity of structures due to the loss of overall structural stability. It is achieved by introducing a supporting system in which an active force is added to the support. A displacement dependent force is considered for this active force. The magnitude and direction of this force is determined by a given rule. This kind of support may be called an active control system or a dynamic reaction system.

We suggest that one of the rules imposed on the reaction of this support may be the rule of a follower force. It means that the direction of the reactive force follows the rotation of a structural element at the support. In a particular case, the reactive force may remain, for example, tangential to the displaced axis of a structural element. Beck (1952) provides a well-known example of such a tangential follower force. He analysed a column, one end of which was rigidly fixed while the other was free and subjected to a compressive load. The direction of the load was always tangential to the displaced column axis at the loading point. His study of small flexural vibrations showed that the critical load is about eight times the Euler load of the same column. This study showed that a non-conservative stability problem must be analysed by means of a dynamic criterion as commonly used static methods of determining critical loads lead to erroneous results as obtained by Pflüger (1950) and Feodos'ev (1953).

The dynamic method proposed by Beck was later widely discussed by, for instance, Bolotin (1963) and Ziegler (1968). Ziegler analysed those cases when the conservative systems could be stabilized or made unstable by non-conservative follower forces not explicitly dependent on time. Historical reviews of the development of the concept of the follower forces were given by Bolotin (1963) and Herrmann (1967). The vibration and stability of elastic systems subjected to follower forces were reviewed and a comprehensive study of the development in this field was also made by Sundararajan (1975). Many researchers have been interested in problems related to follower forces. Leipholz (1980) describes selected non-conservative problems of stability including the effects of damping, load and mass distribution and support conditions.

For the present study, the most important fact is that non-conservative follower forces not explicitly dependent on time have, in general, a stabilizing effect on structures. In addition to the column studied by Beck, this fact was demonstrated by Argyris and Symeonidis (1981) through numerical examples on frames. Given that one of the inherent features of follower forces is the capability to improve the carrying capacity of a structure, by analogy, we can predict similar results for follower reactive forces. The results presented in the following parts confirm our expectations. In normal engineering practice, a designer has, in principle, no choice of loads acting on a structure. From this point of view, the increase of the carrying capacity by the follower force is meaningless. On the other hand the designer can always design supports in the way he wants. The stabilizing effect of the follower force may be turned to advantage if we impose such force systems on supports. With this system, we could considerably improve the stability of structures. The effect of the proposed support would be worth studying and the results of this preliminary study are presented in this paper.

2. CLASSIFICATIONS AND DEFINITIONS

In this paper we deal with conservative loads, non-conservative follower forces and non-working exceptionally conservative reactive forces. The terminology and classifications used herein were adopted from works by Ziegler (1968) and Argyris and Symeonidis (1981). The classification of conservativeness of a force is based on the condition that a potential function is present for the force. There is no potential function present for the non-working rective forces, but following the existing terminology in the literature we call them exceptionally conservative non-working reactive forces (Argyris and Symeonidis, 1981).

Additionally, we introduce a non-conservative rotation-dependent reactive force applied at a support. To simplify the presentation, we consider a roller support of in-plane

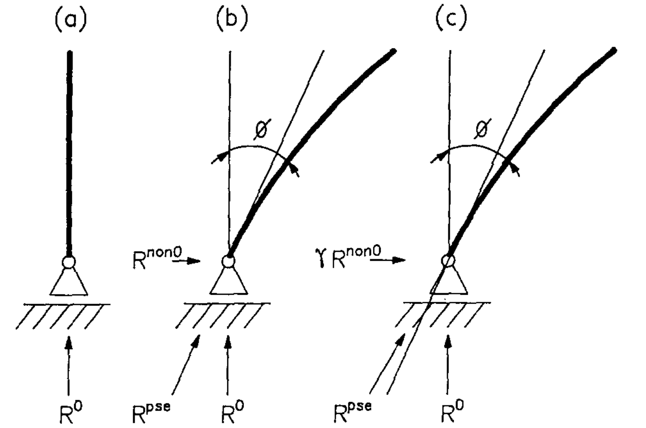

Fig. 1. Roller support: (a) Before the occurrence of instability; (b) After the rotation with tangential pseudo-follower reactive force; (c) After the rotation with pseudo-follower reactive force modified by factor γ.

structures as shown in Fig. 1, for which an active force is applied in the freely movable direction of the support. The roller support with non-working exceptionally reactive force is shown in Fig. 1(a). After the rotation of the support caused by load, the extra force is added to the support. Then, the reactive force consists of two components. One component is the non-working exceptionally conservative reactive force R^0 and the other is an extra force applied at the support $R^{\mathrm{non}0}$. To emphasize that the reactive force includes the rotation-dependent non-conservative force, this reaction is called a pseudo-follower reactive force. The extra rotation-dependent non-conservative force is called the corrective force of the pseudo-follower reactive force. The corrective force controls the direction and the magnitude of the resultant. In a particular case, the resultant R^{pse} of these two forces is tangential to the displaced axis by angle ϕ as shown in Fig. 1(b). In this case the resultant is called a tangential pseudo-follower reactive force. The corrective force of the tangential pseudo-follower reactive force is written as $R^{\mathrm{non}0}(R^0, \phi) = R^0\phi$ when the rotation is sufficiently small. To increase the critical load, it is not necessary for the direction of the follower force to be tangential. To consider a non-tangential pseudo-follower reactive force, a constant multiplication factor γ is introduced to the particular corrective force which makes the direction of the reaction tangential. Then, the corrective force can be expressed as

$$\gamma R^{\mathrm{non}0}(R^0, \phi) = \gamma R^0 \phi. \tag{1}$$

This case is shown in Fig. 1(c). When $\gamma = 0$, the corrective force is equal to zero and the pseudo-follower reactive force is reduced to the ordinary reaction of a roller support. With the increase or decrease of γ, the direction of the reaction changes. When $\gamma = 1$, it becomes a tangential pseudo-follower reactive force.

Argyris and Symeonidis (1981) classified systems with respect to the form of instability as: purely conservative systems; divergence type non-conservative systems; flutter type non-conservative systems; and hybrid-type systems. After the addition of the corrective force at a support, we can still utilize the above for the classification of the systems.

3. FORMULATION OF STABILITY WITH DISPLACEMENT-DEPENDENT REACTIONS

The governing equilibrium equations, strain-displacement relations and elastic constitutive equations for the elastic finite displacement theory for a three-dimensional solid are given for example by Washizu (1982). According to his work, the critical value of an elastic stability problem can be determined by checking the existence of at least one additional, distinct equilibrium in the very close neighborhood of an equilibrium configuration under the identical loading. The determination of the adjacent equilibrium can be made by the linearized theory at the original equilibrium configuration which we call a reference state. The criterion given above needs to be modified for the non-conservative system. For this system, the form of equilibrium is said to be stable if a slight disturbance causes a small deviation of the system from the considered equilibrium configuration, but by decreasing the magnitude of the disturbance the deviation can be made as small as required. On the other hand, the equilibrium is said to be unstable if a disturbance, however small, causes a finite deviation of the system from the considered form of equilibrium.

The formulation presented by Washizu (1982) can be modified to incorporate the above written criterion. The second Piola–Kirchhoff stress tensor, the Green's strain tensor, displacements, body and surface forces are denoted with the addition of the nought symbol $(\ )^0$ as σ_{ij}^0, e_{ij}^0, u_i^0, P_i^0 and F_i^0. Both conservative and non-conservative follower forces are considered for both body and surface forces. For a design analysis, these quantities are usually the quantities under the design load. With the assumption (Washizu, 1982) that changes in geometrical configuration of the body remain negligible until the instability occurs, all the responses of the body caused by external forces are linear. Because of the linear response, when forces are increased monotonically λ times from the loading condition, the body is in equilibrium with stresses $\lambda\sigma_{ij}^0$ under body forces λP_i^0 in volume V, and surface traction λF_i^0 and the exceptionally conservative non-working reactive forces λR_i^0 on the surface S. This equilibrium state is regarded as the reference state. In the adjacent equilibrium configuration under the action of the identical magnitudes of non-conservative forces but without the addition of the corrective forces, we can write the stresses, strains, body and surface forces as $\lambda\sigma_{ij}^0+\sigma_{ij}$, $\lambda e_{ij}^0+e_{ij}$, $\lambda P_i^0+\lambda P_i^{\mathrm{non}0}$, $\lambda F_i^0+\lambda F_i^{\mathrm{non}0}+\lambda R_i^{\mathrm{non}0}$, where $(\ )^{\mathrm{non}}$ = quantities arising due to displacement-dependency of loads including the corrective forces, and the symbol "non" stands for non-conservativeness; P_i = body forces; F_i = surface forces; and R_i = reactive forces working at support; $P_i^{\mathrm{non}0}$ and $F_i^{\mathrm{non}0}$ = linear components of forces at displaced state produced by displacement-dependency of follower forces; and $R_i^{\mathrm{non}0}$ = corrective force of the pseudo-follower reactive force. Both R_i and F_i are external surface forces applied at the surface but distinct notations are used to emphasize the difference between the corrective forces of the pseudo-follower reactive forces and the ordinary surface forces. Since the existence of a distinct equilibrium in a very close neighborhood is of concern, the incremental quantities are of infinitesimal magnitudes, while the quantities at the reference state are of finite magnitudes.

To study instability using the small oscillation method, the virtual work equation for an adjacent equilibrium (Washizu, 1982) is supplemented with the term representing the work done by d'Alambert's force. Supplementing also the terms arising due to displacement-dependency of the forces and the corrective force, it can be written as

$$\int_V (\lambda\sigma_{ij}^0+\sigma_{ij})\delta(\lambda e_{ij}^0+e_{ij})\,\mathrm{d}V-\int_V(\lambda P_i^0+\lambda P_i^{\mathrm{non}0})\delta u_i\,\mathrm{d}V$$
$$-\int_S(\lambda F_i^0+\lambda F_i^{\mathrm{non}0}+\lambda\gamma_i R_i^{\mathrm{non}0})\delta u_i\,\mathrm{d}S+\int_V(\rho_{\mathrm{m}}\ddot{u}_i)\delta u_i\,\mathrm{d}V=0, \quad (2)$$

where ρ_{m} = mass density of material, $(\ddot{\ })$ = double differentiation with respect to time and

$$(\lambda e_{ij}^0+e_{ij}) = \tfrac{1}{2}\big((\lambda u_i^0+u_i)_{,j}+(\lambda u_j^0+u_j)_{,i}+(\lambda u_k^0+u_k)_{,i}(\lambda u_k^0+u_k)_{,j}\big). \quad (3)$$

Note that the term $R_i^{\mathrm{non}0}$ is multiplied by λ because $R_i^{\mathrm{non}0}$ is described as expressed in eqn (1). The integration is performed over the volume V and surface S before the deformation of the body.

Noting that the reference state is in static equilibrium, a similar virtual work equation holds in the reference state except for the term representing work done by d'Alambert's force and those of the displacement-dependent forces. The difference between eqn (2) and that at the reference state and the negligence of the higher order terms of small quantities result in

$$\int_V (\sigma_{ij}\delta\varepsilon_{ij}+\lambda\sigma^0_{ij}u_{k,i}\delta u_{k,j})\,\mathrm{d}V-\int_V (\lambda P^{\mathrm{non0}}_i)\delta u_i\,\mathrm{d}V$$

$$-\int_S (\lambda F^{\mathrm{non0}}_i+\lambda\gamma_i R^{\mathrm{non0}}_i)\delta u_i\,\mathrm{d}S+\int_V (\rho_{\mathrm{m}}\ddot{u}_i)\delta u_i\,\mathrm{d}V=0, \quad (4)$$

where

$$\varepsilon_{ij}=\tfrac{1}{2}(\delta_{ik}+\lambda u^0_{k,i})u_{k,j}+(\delta_{jk}+\lambda u^0_{k,j})u_{k,i} \quad \text{and} \quad \sigma_{ij}=E_{ijkl}\varepsilon_{kl}. \quad (5,6)$$

Since incremental quantities from the reference state are of infinitesimal magnitudes, stresses σ_{ij}, forces P, F and R in eqn (4) are linear functions of the incremental displacement u_i and their derivatives.

The governing equilibrium equations and associated boundary conditions can be obtained from eqn (4).

4. CORRECTIVE MATRIX

Let us consider the case for which a non-working exceptionally conservative reactive force λR^0_i is acting at the reference state at a node i. When the reactive force has two components in x and y directions in a global coordinate system, these components are expressed by λR^0_{xi} and λR^0_{yi}. The component of the rotational displacement vector perpendicular to the x–y plane is denoted by ϕ_{zi}. Although the magnitude of the exceptionally conservative non-working reactive force does not change its magnitude during instability motion, the magnitudes of the components of the pseudo-follower reactive force change by the application of the corrective force $\lambda R^{\mathrm{non0}}_i$. The components of the pseudo-follower reactive force after displacement are given by the resultants of the components of the corrective force and the non-working exceptionally conservative reactive force. With the assumption that $\phi_{zi} \ll 1$, they are given by $\lambda R^0_{xi}+\lambda\gamma R^{\mathrm{non0}}_{yi}$ and $\lambda R^0_{yi}+\lambda\gamma R^{\mathrm{non0}}_{xi}$. The difference between the new components and components at the reference state can, then, be written as

$$\lambda\{R^{\mathrm{non0}}_i\}=\lambda\gamma\{\mathbf{K}_{\mathrm{CR}i}\}\{d_i\}, \quad (7)$$

where

$$\{R^{\mathrm{non0}}_i\}=\langle R^{\mathrm{non0}}_{xi},R^{\mathrm{non0}}_{yi},0\rangle^{\mathrm{T}}=\langle -R^0_{yi}\phi_{zi},R^0_{xi}\phi_{zi},0\rangle^{\mathrm{T}}, \quad (8)$$

$$\{d_i\}=\langle u_{xi},u_{yi},\phi_{zi}\rangle^{\mathrm{T}} \quad (9)$$

and $\mathbf{K}_{\mathrm{CR}i}$ for the ith node is expressed in an explicit form as

$$\mathbf{K}_{\mathrm{CR}i}=\begin{bmatrix} 0 & 0 & -R^0_{yi} \\ 0 & 0 & R^0_{xi} \\ 0 & 0 & 0 \end{bmatrix}. \quad (10)$$

We can define $\mathbf{K}_{\mathrm{CR}}$ as

$$\mathbf{K}_{\mathrm{CR}}=\mathrm{diag}\,[\gamma_1\mathbf{K}_{\mathrm{CR}1},\ldots,\gamma_i\mathbf{K}_{\mathrm{CR}i},\ldots,\gamma_N\mathbf{K}_{\mathrm{CR}N}], \quad (11)$$

where N = number of nodes where corrective forces act in the system; and diag [...] = $N\times D$ dimensional diagonal matrix, D being number of degrees of freedom per node. In the case of in-plane framed structures, $D=3$. Following the same derivation but replacing R^0 by F^0 and R^{non0} by F^{non0}, for which the identical relation to eqn (8) holds, we could obtain the corrective matrix for nodal active tangential follower forces.

5. CRITERIA FOR STABILITY PROBLEM

The limit of the stability of a system is given by the singularity condition of the following matrix, which can be obtained from eqn (4) by using a standard finite element integration technique of the governing differential equations after imposing kinematic constraints such as the Bernouli–Euler hypothesis for the case of beam elements

$$\det[\mathbf{K}_E+\lambda\{\mathbf{K}_G(Q^0)+\mathbf{K}_{CF}(F^0)+\mathbf{K}_{CP}(P^0)-\mathbf{K}_{CR}(\gamma,R^0)\}-\omega^2\mathbf{M}]=0, \tag{12}$$

in which ω is the frequency introduced by $\{d\} = \{f\}\,e^{i\omega t}$ and $\{f\}$ = amplitude independent of time t; P^0 and F^0 = displacement-dependent distributed loads and nodal forces at the reference state, respectively; R^0 = exceptionally conservative non-working reactive forces; Q^0 = stress resultant under the design loading; $\mathbf{K}_E$ = elastic stiffness matrix of small displacements; $\mathbf{K}_G$ = geometric stiffness matrix; $\mathbf{K}_{CF}$, $\mathbf{K}_{CP}$ = corrective matrices for nodal and distributed loads; and $\mathbf{K}_{CR}$ = corrective matrix for nodal reactive forces applied at the support. The matrices $\mathbf{K}_G$, $\mathbf{K}_{CF}$, $\mathbf{K}_{CP}$ and $\mathbf{K}_{CR}$ are functions of the applied loads.

After the addition of the corrective forces at supports, we can find the load multiplier λ for which this structure becomes unstable from the solution of eqn (12) using the same method as used by Argyris and Symeonidis (1981). Equation (12) is solved for the smallest absolute value of λ for which ω is either equal to zero or becomes a complex number with a negative imaginary part. The first case corresponds to static instability (divergence) and the other to dynamic instability (flutter). The sign of λ depends on how loads are applied. The critical values we want can be obtained as λF^0 and λP^0.

6. NUMERICAL EXAMPLES

The phenomenon caused by the addition of the newly introduced supports with the pseudo-follower reactive force is illustrated through numerical examples of in-plane framed structures. The difference of the carrying capacity due to the difference of the loading and supporting systems are studied for a variety of these combinations. The geometric stiffness matrix available in the literature as given for example by Przemieniecki (1968) was used in the following analysis. This geometric matrix can be derived from eqn (4) considering only internal axial forces, which cause axial stresses and displacements, at the reference state.

The frame shown in Fig. 2 is investigated first for the following four cases. The frame is subjected to: (a) a conservative concentrated force with the support being the ordinary roller support subjected to exceptionally conservative non-working reactive force [Fig. 2(a)]; (b) a tangential follower force with the same support as in case (a) [Fig. 2(b)]; (c) a

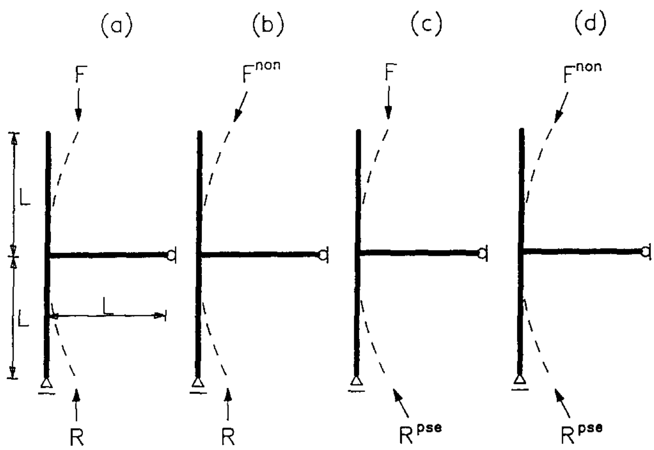

Fig. 2. In-plane frame: (a) Conservative load and roller support; (b) Tangential follower force and roller support; (c) Conservative force and support with tangential pseudo-follower reactive force; (d) Tangential follower force and support with tangential pseudo-follower reactive force.

Table 1. Comparison of ratios of critical loads of frames in Fig. 2 (a), (b), (c), (d) with the critical load of the frame in case (a)

Case	(a)	(b)	(c)	(d)
Load	conservative force	tangential follower force	conservative force	tangential follower force
Support	roller	roller	tangential pseudo follower	tangential pseudo follower
Type of instability	divergence	divergence	divergence	flutter
Ratio	1	1.46	1.46	16.62

conservative force with the roller support subject to a tangential pseudo-follower reactive force [Fig. 2(c)]; and (d) a tangential follower force with the same support as in case (c) [Fig. 2(d)]. The classifications of loadings, the type of instability and the ratios of the critical loads with a reference to case (a) are given in Table 1. In cases (b) and (c), the increment ratios of the critical loads are equal to 1.46. The same ratios are due to the symmetry of the frame. The significant effect is seen in case (d). The implementation of the proposed support with the tangential pseudo-follower reactive force increases the carrying capacity more than 16 times. The results show a significant increase of the carrrying capacity of the system due to the proposed support.

Another example is shown in Fig. 3 where the frame is subject in all four cases to a conservative external force. Each frame possesses different kinds of supports, namely, an ordinary roller support, a roller support with the tangential pseudo-follower reactive force, a hinge support and a fixed support. The results are listed in Table 2. Compared with the critical value of the frame with the roller support, the increase in the carrying capacity for the hinge support is 9.8 times, while the proposed support with the tangential pseudo-follower reactive force is 11.3 times. In the case of the fixed support, the increase of the carrying capacity is the largest at 19.03 times.

To observe the changes of the critical values with the change of the direction of the pseudo-follower reactive force, the direction of the reactive force is changed by multiplying the corrective force by the factor γ introduced in eqn (1). The results are plotted in Fig. 4. The carrying capacity was non-dimensionalized by the critical load of the frame with a roller support. A square symbol represents a critical load due to flutter and a cross symbol due to divergence. The critical load increases with the increase of γ. When $\gamma = 0$, it coincides with the critical load of the ordinary roller support. When γ is negative, the critical load is even less than that with the roller support. The jump of the critical load can be seen between $\gamma = 0$ and 1. This sudden increase of the critical load is due to the change of the loss of the stability from divergence to flutter. The tangential pseudo-follower reactive force of Fig. 3

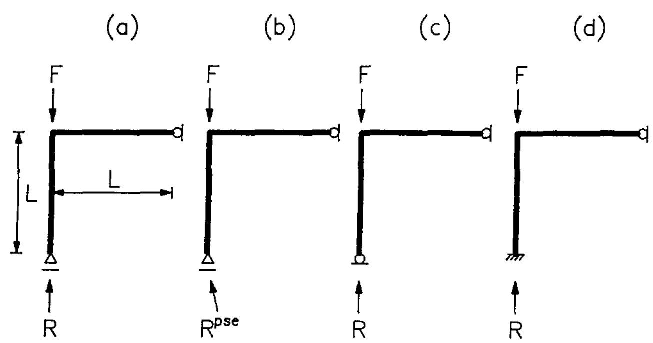

Fig. 3. Frame under conservative load: (a) Roller support; (b) Support with tangential pseudo-follower reactive force; (c) Hinge support; (d) Fixed support.

Table 2. Comparison of ratios of critical loads of frames in Fig. 3 (a), (b), (c), (d) to the critical load of the frame in case (a)

Case	(a)	(b)	(c)	(d)
Load	conservative force	conservative force	conservative force	conservative force
Support	roller	tangential pseudo follower	hinge	fixed
Type of instability	divergence	flutter	divergence	divergence
Ratio	1	11.3	9.8	19.03

corresponds to the case of $\gamma = 1$. For γ around 1.3 and above, the critical load exceeds that for the fixed support shown in Fig. 3(d).

The fixed support can be represented by a roller support with infinitely stiff springs for translation and rotation. The fact that the carrying capacity exceeds that for the fixed support for certain values of γ leads to the conclusion that the proposed support increases the stability more than the stiffest spring and cannot be identified with an ordinary spring. There is a certain limit in the increase of the carrying capacity which occurs around $\gamma = 2$. With a further increase in γ, the decrease of the carrying capacity is observed, as marked by cross symbols. This is due to the change of the stability failure from flutter to divergence.

There may be a possibility that other functions can be imposed on the corrective force of the pseudo-follower reactive force. We can think, for example, about displacements in other parts of the structure. In this paper, however, our attention is confined to the corrective force of the rotation dependent pseudo-follower reactive force at the support where the corrective force is applied.

7. REALIZABILITY

The question remains whether it is possible to realize the proposed support, until careful study on the realizability is made. The following, however, suggests one possible way to realize the system.

Noting the fact that hinge and fixed supports are widely used in the construction of framed structures, it is not a difficult task to construct a device to resist the horizontal reaction. Our consideration is limited to a static problem or a problem for which the change of loading with time is slow so that the problem can be solved as a static problem with different loading positions. For this case, the corrective force of the pseudo-follower reactive force $\lambda\gamma R^{\text{non}0}$ could be applied by an oil jack with a motor controlled valve. For this purpose, the valve is to be controlled by a sensor system, which is attached to the end of the structural

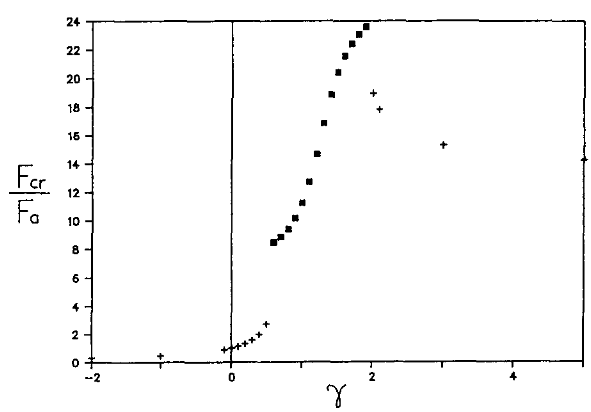

Fig. 4. Dependence of the carrying capacity of the frame in Fig. 3(b) on the modification factor γ.

element at the support and can measure both the rotational angle and the value of the exceptionally conservative non-working reactive force. As can be seen in Fig. 4, it is not necessary for the control of the corrective forces to be very exact to achieve the designated direction. A certain error in its direction and magnitude would be tolerated by shifting the direction to the safer side.

Since the carrying capacity increases monotonically with the increase of the multiplying factor γ within a certain limit, a slightly larger value of corrective force than the optimum value would be preferable at least in the case of the example in Fig. 4 for the above mentioned safety. The increase of the value of the corrective force also increases the cost of the system to apply the force. This leads to an optimization problem to decide the value of the optimal corrective force. Since the cost of construction of the system to apply the corrective force may not be much different for a small difference in the magnitude of the force, no study is made for this optimization.

8. POSSIBLE APPLICATIONS

Assuming that the support with the pseudo-follower reactive force is realizable, we would like to suggest some possible applications to encourage future study on pseudo-follower reactive force. The main emphasis of the proposal is for the design of permanent support, even if this cannot be realized in the near future. More immediate and possible applications of the proposed system would be the temporary improvement of carrying capacity. We could think about larger structures with fewer supports.

The following fields would be the first step of the application of the proposed system. (1) It could be used during an assembly process when the incomplete structure needs a temporary improvement in the carrying capacity. During the erection of the structures, not only the structural shapes but also both loading conditions and positions of supports, i.e. the boundary conditions, are usually significantly different compared with their completed conditions. There are many members which are determined by the condition during the erection rather than by the loading after the completion of the structures. (2) The system could be useful in temporarily improving the carrying capacity of a structure where the increase of the load is rare but predictable, for example transportation of heavy equipment over an existing bridge. (3) The system could also be useful to improve the safety of existing structures, which either may have lost the carrying capacity or have to carry increased loading compared with the design load, or when modification is desired, for example, the addition of one storey. (4) The system could be used to improve the carrying capacity for very large loadings but their occurrence is scarce, such as earthquakes and heavy wind. However, the reliability of the proposed supports is subject to question.

9. CONCLUSIONS

We presented one possible way to improve the carrying capacity of structures in the application of the additional displacement-dependent forces at supports. This improvement of the carrying capacity for the failure due to the loss of stability was numerically studied. In the numerical examples of this report, only the corrective force for a roller support of in-plane structures was studied. There may be, however, other ways to apply additional forces, which could be the subject of a further study.

We found the critical load under the assumption that, in spite of the increase of loading, changes in geometrical configuration of the body remain negligible until the instability occurs. Reality may be different, however, and obtained results should be verified by a more exact non-linear approach. Initial imperfections and other factors should be considered.

The concept of the pseudo-follower reactive force can easily be extended. The corrective force may be understood as either ordinary force or moment and similarly, displacement components can be understood as both rotations and displacements. The rule according to which the reactive force is applied may be arbitrary provided that it increases carrying capacity.

The proposed method indicates a possible improvement in the carrying capacity of a system, which is not reported in the literature checked by the authors.

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